

Ex. 9.

6 - a) Soit $z = (x, y) \in D(z_0; \frac{1}{2}|z_0|)$. On a

$$|z - z_0| < \frac{1}{2}|z_0| = \frac{1}{2}|x_0|. \text{ Comme } |\operatorname{Re}(z - z_0)| \leq |z - z_0|, \text{ on a}$$

$$|x - x_0| = |\operatorname{Re}(z - z_0)| < \frac{1}{2}|x_0| \text{ et } x_0 - \frac{1}{2}|x_0| < x < x_0 + \frac{1}{2}|x_0|.$$

$$\text{Comme } x_0 < 0, \quad x \leq x_0 + \frac{1}{2}|x_0| = x_0 - \frac{1}{2}x_0 = \frac{x_0}{2} < 0.$$

$$\text{Dne } x < 0 \text{ et } |x| = -x \geq -\frac{x_0}{2} = \frac{|x_0|}{2}. \text{ On a donc}$$

$$\left| \frac{y}{x} \right| \leq |y| \frac{2}{|x_0|}.$$

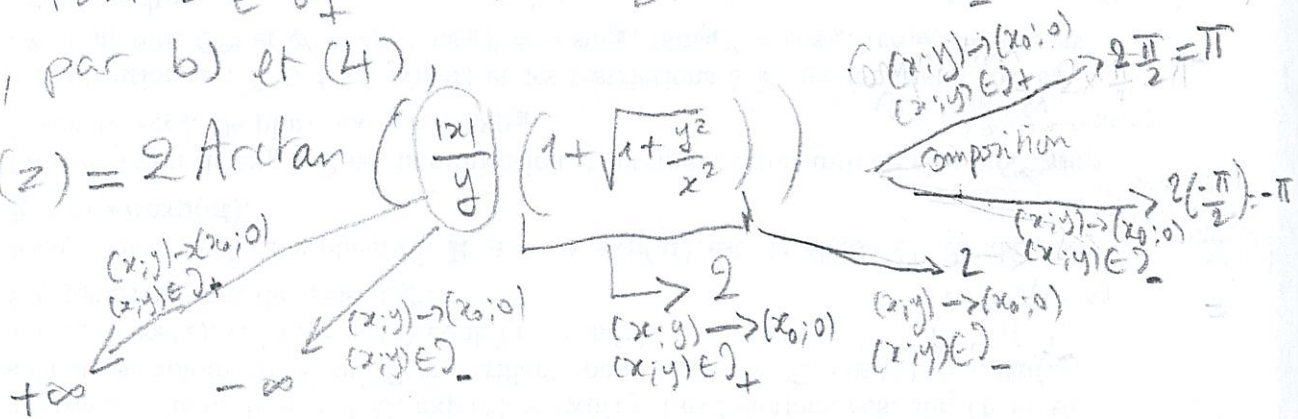
b) Par a), on sait que $x < 0$. Comme $y \neq 0$, on a

$$\frac{y}{x + \sqrt{x^2 + y^2}} = \frac{y}{x + |x| \sqrt{1 + \frac{y^2}{x^2}}} = \frac{y}{-|x| + |x| \sqrt{1 + \frac{y^2}{x^2}}} = \frac{y/|x|}{\sqrt{1 + \frac{y^2}{x^2}} - 1} = \frac{y}{|x|} \frac{\sqrt{1 + \frac{y^2}{x^2}} + 1}{1 + \frac{y^2}{x^2} - 1}$$

$$= \frac{y}{|x|} \frac{|x|^2}{y^2} \left(1 + \sqrt{1 + \frac{y^2}{x^2}} \right) = \frac{|x|}{y} \left(1 + \sqrt{1 + \frac{y^2}{x^2}} \right).$$

c). Pour $z \in \mathcal{I}_+ \cap D(z_0; \frac{|z_0|}{2})$ ou $z \in \mathcal{I}_- \cap D(z_0; \frac{|z_0|}{2})$, on a, par b) et (4),

$$\operatorname{Arg}(z) = 2 \operatorname{Arctan} \left(\frac{|x|}{y} \left(1 + \sqrt{1 + \frac{y^2}{x^2}} \right) \right)$$



$$\text{Car } \lim_{t \rightarrow +\infty} \operatorname{Arctan} t = \frac{\pi}{2} \text{ et } \lim_{t \rightarrow -\infty} \operatorname{Arctan} t = -\frac{\pi}{2}.$$